

Calculators and mobile phones are not allowed

Answer all of the following questions

1. (a) Let $f(x) = \ln(1 + \sqrt{x})$; $x > 0$. Show that f is one-to-one, and find its inverse f^{-1} .
State the domain and range of f^{-1} . (4 pts)

(b) Show that $(\sinh x + \cosh x)^n = \sinh nx + \cosh nx$. (4 pts)

2. Evaluate the following integrals: (4 pts each)

(a) $\int x \sec^{-1} x \, dx$

(b) $\int \frac{\cot^5 x}{\sin x} \, dx$

(c) $\int \frac{3x^2 + x + 3}{(2x - 1)(x^2 + 4)} \, dx$

(d) $\int \frac{dx}{(x^2 + 4x + 13)^{3/2}}$

3. (a) Find the limit $\lim_{x \rightarrow 0^+} (e^{2x} - 1)^{1/\ln x}$. (4 pts)

(b) Determine whether the integral $\int_0^{\infty} \frac{dx}{e^x + e^{-x}}$ is convergent or divergent.
If convergent, find its value. (4 pts)

4. (a) Find the centre, vertices, foci and the equations of the asymptotes of the following hyperbola, then sketch its graph. (5 pts)

$$4x^2 - 25y^2 + 50y + 8x - 121 = 0.$$

(b) Find the area of the region outside $r = 2(1 + \cos \theta)$ and inside $r = 6 \cos \theta$. (5 pts)

5. (a) Find the parametric equations of the line of intersection of the planes (4 pts)

$$\pi_1: 3x + 4y + 2z = 4 \quad \text{and} \quad \pi_2: 2x + 3y - z = 1$$

(b) Find an equation of the plane passing through the point $P(7, 0, 3)$ and containing the line (4 pts)

$$\frac{x}{1} = \frac{y-4}{1} = \frac{z-6}{-2}$$